Spring 2017 - Math 689
Calculus of Variations

Lectures: M 7:20pm-10:00pm

Instructor: Harbir Antil
Office: Exploratory Hall, room 4201
Phone: (703) 993-5777
Email: hantil@gmu.edu
Webpage: `http://math.gmu.edu/~hantil`

Prerequisites. Functional analysis and PDE theory (variational method) will be reviewed. Prior exposure to graduate level PDE and MATLAB is useful but not mandatory.

Course description: Every process in nature tries to optimize something under the available resources. The limited available resources act as constraints and, for this reason, constrained optimization problems are ubiquitous in science and engineering. A systematic approach is needed to allocate the resources, without violating the constraints and still being effective in minimizing the cost. Due to the nonsmoothness the constraints introduce, the optimization problems are nonlinear. Moreover, many of the partial differential equations that describe the constraints are geometric, nonlinear, multiscale with an unknown domain, i.e. free boundary problems. This makes the development and analysis of numerical techniques for the solution of these problems extremely challenging and of practical relevance. This course consists of

Selected topics in Calculus of Variations*. Suggested text [1, 2, 4, 5].

Geometric PDE on Deformable Surfaces*. Suggested text [7, 3, 6].

- Representation of parametrized surfaces.
- Differential operators on surfaces.
- Discretization of Laplace-Beltrami operator.
- Shape differential calculus.
- Shape derivatives of geometric quantities and functionals.

*Handwritten notes will be provided.

Exams (30%): The FINAL exam / project will be comprehensive and will constitute 30% of the grade.

Homework (70%): There will be about 6 HOMEWORKS which will amount to 70% of the final grade. The homeworks will be about 80% theoretical and 20% computational using MATLAB. There will be a penalty of 10% per day late; homeworks will not be accepted after one week.

Students are encouraged to work in groups of up to three students but must hand in an individual self written proofs and answers.

Academic Integrity: GMU is an Honor Code university; please see the Office for Academic Integrity for a full description of the code and the honor committee process. The principle of academic integrity is taken very seriously and violations are treated gravely. What does academic integrity mean in this course? Essentially this: when you are responsible for a task, you will perform that task. When you rely on someone elses work in an aspect of the performance of that task, you will give full credit in the proper, accepted form. Another aspect of academic integrity is the free play of ideas. Vigorous discussion and debate are encouraged in this course, with the firm expectation that all aspects of the class will be conducted with civility and respect for differing ideas, perspectives, and traditions. When in doubt (of any kind) please ask for guidance and clarification.

Mason email accounts. Students must use their MasonLIVE email account for any correspondence during this course. For more information see: `http://masonlive.gmu.edu`.

Office of Disability Services. If you are a student with a disability and you need academic accommodations, please see me and contact the Office of Disability Services (ODS) at 993-2474, `http://ods.gmu.edu`. All academic accommodations must be arranged through the ODS.

University policies The University Catalog, `http://catalog.gmu.edu`, is the central resource for university policies affecting student, faculty, and staff conduct in university academic affairs. Other policies are available at `http://universitypolicy.gmu.edu`. All members of the university community are responsible for knowing and following established policies.

References

[1] G. Buttazzo, M. Giaquinta, and S. Hildebrandt. *One-dimensional variational problems*, volume 15 of *Oxford Lecture Series in Mathematics and its Applications*. The Clarendon Press, Oxford University Press, New York, 1998. An introduction.

[2] B. Dacorogna. *Direct methods in the calculus of variations*, volume 78 of *Applied Mathematical Sciences*. Springer, New York, second edition, 2008.

[3] M. C. Delfour and J.-P. Zolésio. Shape analysis via distance functions: local theory. In *Boundaries, interfaces, and transitions (Banff, AB, 1995)*, volume 13 of *CRM Proc. Lecture Notes*, pages 91–123. Amer. Math. Soc., Providence, RI, 1998.

[4] I. Ekeland and R. Témam. *Convex analysis and variational problems*, volume 28 of *Classics in Applied Mathematics*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, english edition, 1999. Translated from the French.

[5] J. Jost and X. Li-Jost. *Calculus of variations*, volume 64 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1998.

[6] B. Mohammadi and O. Pironneau. *Applied shape optimization for fluids*. Numerical Mathematics and Scientific Computation. Oxford University Press, Oxford, second edition, 2010.

[7] J. Sokołowski and J. P. Zolésio. *Introduction to shape optimization*, volume 16 of *Springer Series in Computational Mathematics*. Springer-Verlag, Berlin, 1992. Shape sensitivity analysis.